Find:

$$\Omega = \int_0^2 \frac{dx}{\sqrt{2x+1} + \sqrt[3]{2x+1}}$$

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$$\Omega = \int_0^2 \frac{dx}{\sqrt{2x+1} + \sqrt[3]{2x+1}} \overset{y^6=2x+1}{=\!=\!=} \int_1^{\sqrt[6]{5}} \frac{3y^5}{y^3 + y^2}\, dy =$$

$$= 3 \int_1^{\sqrt[6]{5}} \frac{y^3}{y+1}\, dy = 3 \int_1^{\sqrt[6]{5}} \frac{(y+1)(y^2 - y + 1) - 1}{y+1}\, dy =$$

$$= 3 \int_1^{\sqrt[6]{5}} (y^2 - y + 1)\, dy - 3 \int_1^{\sqrt[6]{5}} \frac{1}{y+1}\, dy =$$

$$= 3 \left(\frac{\sqrt[6]{125}}{3} - \frac{1}{3} - \frac{\sqrt[6]{25}}{2} + \frac{1}{2} + \sqrt[6]{5} - 1 \right) = 3 \left(\frac{\sqrt[6]{125}}{3} - \frac{\sqrt[6]{25}}{2} + \sqrt[6]{5} - \frac{5}{6} \right)$$