ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^1 \frac{1}{1 + \sqrt[3]{2x+1}}\,dx$$

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Solution by Daniel Sitaru-Romania

$$\Omega = \int_0^1 \frac{1}{1+\sqrt[3]{2x+1}}\,dx \overset{y^3=2x+1}{=} \frac{3}{2}\int_1^{\sqrt[3]{3}} \frac{y^2}{1+y}\,dy = \frac{3}{2}\int_1^{\sqrt[3]{3}} \frac{y^2-1+1}{1+y}\,dy =$$

$$= \frac{3}{2}\int_1^{\sqrt[3]{3}} (y-1)\,dy + \frac{3}{2}\int_1^{\sqrt[3]{3}} \frac{1}{1+y}\,dy =$$

$$= \frac{3}{2}\left(\frac{\sqrt[3]{9}}{2} - \frac{1}{2} - \sqrt[3]{3} + 1\right) + \frac{3}{2} \ln\left(\frac{1+\sqrt[3]{3}}{2}\right) = \frac{3}{4}\left(\sqrt[3]{9} - \sqrt[3]{3} + 1\right) + \frac{3}{2} \ln\left(\frac{1+\sqrt[3]{3}}{2}\right)$$