

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^1 \frac{1}{1 + \sqrt{2x + 1}} dx$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \Omega &= \int_0^1 \frac{1}{1 + \sqrt{2x + 1}} dx \stackrel{y^2 = 2x + 1}{\cong} \int_1^{\sqrt{3}} \frac{y}{1 + y} dy = \int_1^{\sqrt{3}} \frac{1 + y - 1}{1 + y} dy = \\ &= \int_1^{\sqrt{3}} dy - \int_1^{\sqrt{3}} \frac{1}{1 + y} dy = \sqrt{3} - 1 - \ln(1 + \sqrt{3}) + \ln 2 = \\ &= \sqrt{3} - 1 + \ln\left(\frac{2}{1 + \sqrt{3}}\right) \end{aligned}$$