ROMANIAN MATHEMATICAL MAGAZINE

**Find:**

$$\int_0^{\frac{\pi}{6}} \frac{1}{\cos(2x)+\cos(x)}dx$$

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$$\Omega = \int_0^{\frac{\pi}{6}} \frac{1}{\cos(2x)+\cos(x)}dx = \int_0^{\frac{\pi}{6}} \frac{1}{2cos^2(x)+\cos(x)-1}dx =$$

$$= \frac{2}{3}\int_0^{\frac{\pi}{6}} \frac{dx}{2\cos(x)-1} - \frac{1}{3}\int_0^{\frac{\pi}{6}} \frac{dx}{\cos(x)+1} = \Omega_1 + \Omega_2$$

$$\Omega_1 = \frac{2}{3}\int_0^{\frac{\pi}{6}} \frac{dx}{2\cos(x)-1} \overset{\tan\left(\frac{x}{2}\right)\to t}{\cong} \frac{2}{3}\int_0^{\tan\left(\frac{\pi}{12}\right)} \frac{\frac{2}{1+t^2}}{2\frac{1-t^2}{1+t^2}-1}dt =$$

$$= \frac{2}{3}\int_0^{\tan\left(\frac{\pi}{12}\right)} \frac{2}{1-3t^2}dt = \frac{4}{3}\cdot\frac{1}{3}\int_0^{\tan\left(\frac{\pi}{12}\right)} \frac{1}{\frac{1}{3}-t^2}dt =$$

$$= \frac{4}{9}\cdot\frac{\sqrt{3}}{2} ln\left|\frac{\frac{1}{\sqrt{3}}+t}{\frac{1}{\sqrt{3}}-t}\right|\Bigg|_0^{\tan\left(\frac{\pi}{12}\right)} = \frac{2\sqrt{3}}{9} ln\left|\frac{\frac{1}{\sqrt{3}}+2-\sqrt{3}}{\frac{1}{\sqrt{3}}-2+\sqrt{3}}\right| = \frac{2\sqrt{3}}{9}\ln(1+\sqrt{3})$$

$$\Omega_2 = \frac{1}{3}\int_0^{\frac{\pi}{6}} \frac{dx}{\cos(x)+1} = \frac{1}{3}\int_0^{\frac{\pi}{6}} \frac{dx}{2cos^2\left(\frac{x}{2}\right)} = \frac{1}{3}\tan\left(\frac{x}{2}\right)\Big|_0^{\frac{\pi}{6}} =$$

$$= \frac{1}{3}\tan\left(\frac{\pi}{12}\right) = \frac{1}{3}(2-\sqrt{3})$$

$$\Omega = \Omega_1 + \Omega_2 = \frac{2\sqrt{3}}{9}\ln(1+\sqrt{3}) + \left(-\frac{1}{3}(2-\sqrt{3})\right)$$

$$Therefore:$$

$$\Omega = \frac{2\sqrt{3}}{9}\ln(1+\sqrt{3}) - \frac{1}{3}(2-\sqrt{3})$$