ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sqrt{1+\cos^2 x}}dx$$

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$$\Omega=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cot x}{\sqrt{1+\cos^2 x}}dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{\cos x}{\sin x\sqrt{1+1-\sin^2 x}}dx\overset{y=\sin x}{\cong}$$

$$=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\frac{1}{y\sqrt{2-y^2}}dy\overset{z=\frac{1}{y}}{\cong}\int_{2}^{\frac{2}{\sqrt{3}}}\frac{-\frac{1}{z^2}}{\frac{1}{z}\sqrt{2-\frac{1}{z^2}}}dz=-\int_{2}^{\frac{2}{\sqrt{3}}}\frac{1}{\sqrt{2z^2-1}}dz=$$

$$=\frac{1}{\sqrt{2}}\int_{\frac{2}{\sqrt{3}}}^{2}\frac{1}{\sqrt{z^2-\frac{1}{2}}}dz=\frac{1}{\sqrt{2}}\left(ln\left(2+\sqrt{4-\frac{1}{2}}\right)-ln\left(\frac{1}{\sqrt{2}}+\sqrt{\frac{4}{3}-\frac{1}{2}}\right)\right)=$$

$$=\frac{1}{\sqrt{2}}\left(ln\left(2+\sqrt{\frac{14}{4}}\right)-ln\left(\frac{1}{\sqrt{2}}+\sqrt{\frac{30}{36}}\right)\right)=$$

$$=\frac{1}{\sqrt{2}}ln\left(\frac{\frac{4+\sqrt{14}}{2}}{\frac{6+\sqrt{30}}{6\sqrt{2}}}\right)=\frac{1}{\sqrt{2}}ln\left(\frac{3\sqrt{2}(4+\sqrt{14})}{6+\sqrt{30}}\right)=\frac{1}{\sqrt{2}}ln\left(\frac{12+3\sqrt{14}}{3\sqrt{2}+\sqrt{15}}\right)$$