

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\sqrt{1 + \cos^2 x}} dx$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned}\Omega &= \int_0^{\frac{\pi}{6}} \frac{\sin x}{\sqrt{1 + \cos^2 x}} dx \stackrel{y=\cos x}{\cong} - \int_1^{\frac{\sqrt{3}}{2}} \frac{1}{\sqrt{1 + y^2}} dy = \\ &= -\ln\left(\frac{\sqrt{3}}{2} + \sqrt{1 + \frac{3}{4}}\right) + \ln(1 + \sqrt{1 + 1^2}) = \\ &= \ln(1 + \sqrt{2}) - \ln\left(\frac{\sqrt{3} + \sqrt{7}}{2}\right) = \ln\left(\frac{2 + 2\sqrt{2}}{\sqrt{3} + \sqrt{7}}\right)\end{aligned}$$