

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\sqrt{1 + \sin^2 x}} dx$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\begin{aligned} \Omega &= \int_0^{\frac{\pi}{6}} \frac{\sin x}{\sqrt{1 + \sin^2 x}} dx = \int_0^{\frac{\pi}{6}} \frac{\sin x}{\sqrt{2 - \cos^2 x}} dx \stackrel{y=\cos x}{=} - \int_1^{\frac{\sqrt{3}}{2}} \frac{dy}{\sqrt{2 - y^2}} = \\ &= -\sin^{-1} \left(\frac{\frac{\sqrt{3}}{2}}{\sqrt{2}} \right) + \sin^{-1} \left(\frac{1}{\sqrt{2}} \right) = \frac{\pi}{4} - \sin^{-1} \left(\frac{\sqrt{6}}{4} \right) \end{aligned}$$