

Find:

$$\Omega = \int_0^{\frac{\pi}{6}} \frac{\cos x}{\sqrt{1 + \sin^2 x}} dx$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\begin{aligned} \Omega &= \int_0^{\frac{\pi}{6}} \frac{\cos x}{\sqrt{1 + \sin^2 x}} dx \stackrel{y=\sin x}{\cong} \int_0^{\frac{1}{2}} \frac{1}{\sqrt{1 + y^2}} dy = \\ &= \ln \left(\frac{1}{2} + \sqrt{1 + \frac{1}{4}} \right) - \ln(0 + \sqrt{1 + 0}) = \ln \left(\frac{1 + \sqrt{5}}{2} \right) \end{aligned}$$