ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{cotx}{\sqrt{1+sin^2x}}dx$$

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$$\Omega=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{cotx}{\sqrt{1+sin^2x}}dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{3}}\frac{cosx}{sinx\sqrt{1+sin^2x}}dx\overset{y=sinx}{\cong}$$

$$=\int_{\frac{1}{2}}^{\frac{\sqrt{3}}{2}}\frac{1}{y\sqrt{1+y^2}}dy\overset{z=\frac{1}{y}}{\cong}\int_{2}^{\frac{2}{\sqrt{3}}}\frac{-\frac{1}{z^2}}{\frac{1}{z}\sqrt{1+\frac{1}{z^2}}}dz=-\int_{2}^{\frac{2}{\sqrt{3}}}\frac{1}{\sqrt{z^2+1}}dz=$$

$$=-ln\left(\frac{2}{\sqrt{3}}+\sqrt{\frac{4}{3}+1}\right)+ln\left(2+\sqrt{2^2+1}\right)=$$

$$=ln(2+\sqrt{5})-ln\left(\frac{2+\sqrt{7}}{\sqrt{3}}\right)=ln\left(\frac{2\sqrt{3}+\sqrt{15}}{2+\sqrt{7}}\right)$$