ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_0^{\frac{\pi}{6}}\frac{tanx}{\sqrt{1+sin^2x}}dx$$

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$$\Omega=\int_0^{\frac{\pi}{6}}\frac{tanx}{\sqrt{1+sin^2x}}dx=\int_0^{\frac{\pi}{6}}\frac{sinx}{cosx\sqrt{1+1-cos^2x}}dx\overset{y=cosx}{\cong}$$

$$=\int_1^{\frac{\sqrt{3}}{2}}\frac{-dy}{y\sqrt{2-y^2}}\overset{z=\frac{1}{y}}{\cong}\int_1^{\frac{2}{\sqrt{3}}}\frac{\frac{1}{z^2}}{\frac{1}{z}\sqrt{2-\frac{1}{z^2}}}dz=\int_1^{\frac{2}{\sqrt{3}}}\frac{1}{\sqrt{2z^2-1}}dz=$$

$$=\frac{1}{\sqrt{2}}\int_1^{\frac{2}{\sqrt{3}}}\frac{1}{\sqrt{z^2-\frac{1}{2}}}dz=\frac{1}{\sqrt{2}}\left(ln\left(\frac{2}{\sqrt{3}}+\sqrt{\frac{4}{3}-\frac{1}{2}}\right)-ln\left(1+\sqrt{1-\frac{1}{2}}\right)\right)=$$

$$=\frac{1}{\sqrt{2}}\left(ln\left(\frac{2}{\sqrt{3}}+\sqrt{\frac{5}{6}}\right)-ln\left(1+\frac{1}{\sqrt{2}}\right)\right)=\frac{1}{\sqrt{2}}ln\left(\frac{\frac{2\sqrt{2}+\sqrt{5}}{\sqrt{6}}}{\frac{1+\sqrt{2}}{\sqrt{2}}}\right)=\frac{1}{\sqrt{2}}ln\left(\frac{2\sqrt{2}+\sqrt{5}}{\sqrt{3}+\sqrt{6}}\right)$$