ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^{\frac{\pi}{6}} \frac{cos^3(x)}{(1+\sin(x))^2}\,dx$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\int_0^{\frac{\pi}{6}} \frac{cos^3(x)}{(1+\sin(x))^2}\,dx = \int_0^{\frac{\pi}{2}} \frac{(1-sin^2(x))}{(1+\sin(x))^2}\,dsin(x) =$$

$$= \int_0^{\frac{\pi}{2}} \frac{(1-sin(x))}{1+\sin(x)}\,dsin(x) \overset{sin(x)\to t}{\hat{=}} \int_0^{\frac{1}{2}} \frac{1-t}{1+t}\,dt =$$

$$= \int_0^{\frac{1}{2}} \frac{2-(1+t)}{1+t}\,dt = 2\int_0^{\frac{1}{2}} \frac{1}{1+t}\,dt - \int_0^{\frac{1}{2}} dt =$$

$$= 2ln|1+t|\Big|_0^{\frac{1}{2}} - t\Big|_0^{\frac{1}{2}} = 2\ln\left(\frac{3}{2}\right) - \frac{1}{2} = \ln\left(\frac{9}{4}\right) - \frac{1}{2}$$