ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_0^{\infty}\frac{dx}{1+x^7}$$

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$$\Omega=\int_0^{\infty}\frac{dx}{1+x^7}\overset{y=x^7}{\cong}\int_0^{\infty}\frac{1}{1+y}\cdot\frac{1}{7\sqrt[7]{y^6}}dy=$$

$$=\frac{1}{7}\int_0^{\infty}\frac{y^{-\frac{6}{7}}}{1+y}dy=\frac{1}{7}\int_0^{\infty}\frac{y^{\frac{1}{7}-1}}{(1+y)^{\frac{1}{7}+\frac{6}{7}}}dy=\frac{1}{7}B\left(\frac{1}{7},\frac{6}{7}\right)=$$

$$=\frac{1}{7}\cdot\frac{\Gamma\left(\frac{1}{7}\right)\cdot\Gamma\left(\frac{6}{7}\right)}{\Gamma\left(\frac{1}{7}+\frac{6}{7}\right)}=\frac{1}{7}\cdot\frac{\Gamma\left(\frac{1}{7}\right)\cdot\Gamma\left(1-\frac{1}{7}\right)}{\Gamma(1)}=\frac{1}{7}\cdot\frac{\pi}{sin\frac{\pi}{7}}=\frac{\pi}{7sin\frac{\pi}{7}}$$