ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_0^{\infty}\frac{dx}{(1+x^4)^2}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\Omega=\int_0^{\infty}\frac{dx}{(1+x^4)^2}\overset{y=x^4}{\cong}\int_0^{\infty}\frac{1}{(1+y)^2}\cdot\frac{1}{4\sqrt[4]{y^3}}dy=$$

$$=\frac{1}{4}\int_0^{\infty}\frac{y^{-\frac{3}{4}}}{(1+y)^2}dy=\frac{1}{4}\int_0^{\infty}\frac{y^{\frac{1}{4}-1}}{(1+y)^{\frac{1}{4}+\frac{7}{4}}}dy=\frac{1}{4}B\left(\frac{1}{4},\frac{7}{4}\right)=$$

$$=\frac{1}{4}\cdot\frac{\Gamma\left(\frac{1}{4}\right)\cdot\Gamma\left(\frac{7}{4}\right)}{\Gamma\left(\frac{1}{4}+\frac{7}{4}\right)}=\frac{1}{4}\cdot\frac{\Gamma\left(\frac{1}{4}\right)\cdot\frac{7}{4}\Gamma\left(\frac{3}{4}\right)}{\Gamma(2)}=$$

$$=\frac{7}{16}\cdot\frac{\Gamma\left(\frac{1}{4}\right)\Gamma\left(1-\frac{1}{4}\right)}{1}=\frac{7}{16}\cdot\frac{\pi}{\sin\frac{\pi}{4}}=\frac{7\pi}{8\sqrt{2}}$$