

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$I = \int_0^1 \int_0^1 \int_0^1 \sqrt{x + 2y + 3z} \, dx \, dy \, dz$$

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Solution by Samuel Ajibade-Nigeria

$$\text{Basic rule: } \int (ax + b)^n \, dx = \frac{1}{a(n+1)} (ax + b)^{n+1} + C$$

$$I = \int_0^1 \int_0^1 \int_0^1 \sqrt{x + 2y + 3z} \, dx \, dy \, dz = \frac{2}{3} \int_0^1 \int_0^1 [(1 + 2y + 3z)^{\frac{3}{2}} - (2y + 3z)^{\frac{3}{2}}] \, dy \, dz$$

$$I = \frac{2}{3} \cdot \frac{1}{5} \int_0^1 [(3 + 3z)^{\frac{5}{2}} - (1 + 3z)^{\frac{5}{2}} - (2 + 3z)^{\frac{5}{2}} - (3z)^{\frac{5}{2}}] \, dz$$

$$I = \frac{2}{15} \cdot \frac{2}{21} (6^{\frac{7}{2}} - 3^{\frac{7}{2}} - 4^{\frac{7}{2}} + 1 - 5^{\frac{7}{2}} + 2^{\frac{7}{2}} + 3^{\frac{7}{2}})$$

$$I = \frac{4}{315} (216\sqrt{6} - 125\sqrt{5} + 8\sqrt{2} - 127)$$