

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_{\sqrt{2}}^{\sqrt{3}} \frac{x^3(x^2 + 3)}{(x^4 - 1)(x^2 + 1)} dx$$

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$$I = \int_{\sqrt{2}}^{\sqrt{3}} \frac{x^3(x^2 + 3)}{(x^4 - 1)(x^2 + 1)} dx$$

$$\text{Let } u = x^2 \rightarrow du = 2x dx \quad u \in [2, 3]$$

$$I = \int_2^3 \frac{u(u+3)}{(u^2-1)(u+1)} \cdot \frac{du}{2} = \frac{1}{2} \int_2^3 \frac{u^2+3u}{(u-1)(u+1)^2} du =$$

$$\frac{1}{2} \int_2^3 \frac{(u+1)^2 + u - 1}{(u-1)(u+1)^2} du = \frac{1}{2} \int_2^3 \left(\frac{1}{u-1} + \frac{1}{(u+1)^2} \right) du =$$

$$\frac{1}{2} \left(\int_2^3 \frac{1}{u-1} du + \int_2^3 \frac{1}{(u+1)^2} du \right) = \frac{1}{2} \left(\ln(u-1) - \frac{1}{1+u} \right) \Big|_2^3 = \frac{\ln(2)}{2} + \frac{1}{24}$$