

ROMANIAN MATHEMATICAL MAGAZINE

Find :

$$\int_0^{\arctan(2)} \frac{3 \tan(x) - 1}{4 \cos^2(x) + \sin^2(x)} dx$$

Proposed by Henry Yang-Bangladseh

Solution by Chew Seong Cheong-Malaysia

$$\begin{aligned} & \int_0^{\arctan(2)} \frac{3 \tan(x) - 1}{4 \cos^2(x) + \sin^2(x)} dx \cdot \frac{\sec^2(x)}{\sec^2(x)} = \\ &= \int_0^{\arctan(2)} \frac{(3 \tan(x) - 1) \sec^2(x)}{4 + \tan^2(x)} dx = \int_0^2 \frac{3t - 1}{4 + t^2} dt = \\ &= \int_0^2 \frac{3t}{4 + t^2} dt - \int_0^2 \frac{1}{4 + t^2} dt = \frac{3}{2} \ln(4 + t^2) - \frac{1}{2} \arctan\left(\frac{t}{2}\right) \Big|_0^2 = \\ &= \frac{3}{2} (\ln(8) - \ln(4)) - \frac{1}{2} \left(\frac{\pi}{4} - 0\right) = \frac{3}{2} \ln(2) - \frac{\pi}{8} \end{aligned}$$

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