ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_0^1 \frac{T_{2n}(x)\cos(ax)}{\sqrt{1-x^2}}dx$$

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Solution by Rana Ranino-Algeria

$$\Omega = \int_0^1 \frac{T_{2n}(x)\cos(ax)}{\sqrt{1-x^2}} \overset{x=\cos(\theta)}{\cong} \int_0^{\frac{\pi}{2}} T_{2n}(\cos(\theta))\cos(a\cos(\theta))d\theta =$$

$$= \int_0^{\frac{\pi}{2}} \cos(2n\theta)\cos(a\cos(\theta))d\theta$$

$$Jacobi - Anger's\ \ expansions\ :$$

$$\cos(a\cos(\theta)) = J_0(a) + 2\sum_{k=1}^{\infty}(-1)^k J_{2k}(a)\cos(2k\theta)$$

$$\Omega = J_0(a) + \underbrace{\int_0^{\frac{\pi}{2}}\cos(2n\theta)d\theta}_{0} + \sum_{k=1}^{\infty}(-1)^k J_{2k}(a)\int_0^{\frac{\pi}{2}} 2\cos(2n\theta)\cos(2k\theta)d\theta$$

$$\int_0^{\frac{\pi}{2}} 2\cos(2n\theta)\cos(2k\theta)d\theta = \int_0^{\pi}\cos(n\theta)\cos(k\theta)d\theta = \frac{\pi}{2}\delta_{nk}$$

$$\frac{\pi}{2}\int_0^1 \frac{T_{2n}(x)\cos(ax)}{\sqrt{1-x^2}}dx = (-1)^n J_{2n}(a)$$