In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} \csc\left(\frac{A}{2}\right)\left(\sqrt{\left(ctg^3\left(\frac{B}{2}\right) + ctg^3\left(\frac{C}{2}\right)\right)} + \sqrt{\left(ctg^3\left(\frac{A}{2}\right) + ctg^3\left(\frac{C}{2}\right)\right)}\right) \geq 12\sqrt{6\sqrt{3}}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Mirsadix Muzefferov-Azerbaijan

$$Let : \ LHS = LHS_1 + LHS_2 + LHS_3$$

$$LHS_1 = \csc\left(\frac{A}{2}\right)\left(\sqrt{\left(ctg^3\left(\frac{B}{2}\right) + ctg^3\left(\frac{C}{2}\right)\right)} + \sqrt{\left(ctg^3\left(\frac{B}{2}\right) + ctg^3\left(\frac{C}{2}\right)\right)}\right) \overset{AM-GM}{\gtrless}$$

$$\csc\left(\frac{A}{2}\right)\left(\sqrt{2\left(ctg\left(\frac{B}{2}\right).ctg\left(\frac{C}{2}\right)\right)^{\frac{3}{2}}} + \sqrt{2\left(ctg\left(\frac{B}{2}\right).ctg\left(\frac{C}{2}\right)\right)^{\frac{3}{2}}}\right) \overset{AM-GM}{\gtrless}$$

$$\csc\left(\frac{A}{2}\right)\left(\sqrt[4]{4\left(ctg\left(\frac{A}{2}\right).ctg\left(\frac{B}{2}\right).ctg\left(\frac{C}{2}\right)\right)^{\frac{3}{2}}.\left(ctg\left(\frac{C}{2}\right)\right)^{\frac{3}{2}}}\right)$$

Analogously , it is calculated for LHS_2 and LHS_3

$$LHS \overset{A-G}{\gtrless} 3\sqrt[3]{LHS_1. \ LHS_2. \ LHS_3} =$$

$$= 3.2\left(64\left(ctg\left(\frac{A}{2}\right).ctg\left(\frac{B}{2}\right).ctg\left(\frac{C}{2}\right)\right)^{\frac{9}{2}}.\left(ctg\left(\frac{A}{2}\right).ctg\left(\frac{B}{2}\right).ctg\left(\frac{C}{2}\right)\right)^{\frac{3}{2}}\right)^{\frac{1}{12}}.$$

$$\left(\csc\left(\frac{A}{2}\right).\csc\left(\frac{B}{2}\right).\csc\left(\frac{C}{2}\right)\right)^{\frac{1}{3}} =$$

$$6\left(2\left(ctg\left(\frac{A}{2}\right).ctg\left(\frac{B}{2}\right).ctg\left(\frac{C}{2}\right)\right)\right)^{\frac{1}{2}}.\left(\csc\left(\frac{A}{2}\right).\csc\left(\frac{B}{2}\right).\csc\left(\frac{C}{2}\right)\right)^{\frac{1}{3}} =$$

$$6\left(2\left(\frac{p}{r}\right)\right)^{\frac{1}{2}}.\left(\frac{4R}{r}\right)^{\frac{1}{3}} \overset{Mitrinovic, \ Euler}{\gtrless} 6(2.3\sqrt{3})^{\frac{1}{2}}.(8)^{\frac{1}{3}} = 12\sqrt{6\sqrt{3}}$$

Equality holds if : $A = B = C$