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In acute ΔABC the following relationship holds:

$$\cot\frac{A}{2}\sqrt{cosA}+\cot\frac{B}{2}\sqrt{cosB}+\cot\frac{C}{2}\sqrt{cosC}>\sqrt{3}$$

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Solution by Tapas Das-India

$$as\ 0<cosA<1\ then\ \sqrt{cosA}>cosA\ (1)$$

$$WLOG\ A\ge B\ge C$$
$$\cot\frac{A}{2}\le\cot\frac{B}{2}\le\cot\frac{C}{2}\ \&\ cosA\le cosB\le cosC$$

$$\cot\frac{A}{2}\sqrt{cosA}+\cot\frac{B}{2}\sqrt{cosB}+\cot\frac{C}{2}\sqrt{cosC}\overset{(1)}{\ge}$$

$$\ge\cot\frac{A}{2}cosA+\cot\frac{B}{2}cosB+\cot\frac{C}{2}cosC\overset{Chebyshev}{\ge}\frac{1}{3}\left(\sum\cot\frac{A}{2}\right)\left(\sum cosA\right)=$$

$$=\frac{1}{3}\cdot\frac{s}{r}\cdot\frac{R+r}{R}>\frac{1}{3}\cdot\frac{s}{r}\cdot\frac{R}{R}\overset{Mitrinovic}{>}\frac{1}{3}\cdot\frac{3\sqrt{3}r}{r}=\sqrt{3}$$