ROMANIAN MATHEMATICAL MAGAZINE

**In $\Delta ABC$ the following relationship holds:**

$$2\sqrt{3}F \le \sum AH.h_a \le \frac{9R^2}{2}$$

***Proposed by Sarkhan Adgozalov-Georgia***

***Solution by Qurban Muellim-Azerbaijan***

$$\sum AH.h_a = \sum_{cyc} 2RcosA \cdot \frac{2F}{a} = \sum_{cyc} 2RcosA \cdot \frac{bcsinA}{2RsinA} =$$

$$= \sum bc.\cos(A) = \sum \left(\frac{b^2 + c^2 - a^2}{2}\right) = \frac{\sum a^2}{2} \ge \frac{4\sqrt{3}F}{2} = 2\sqrt{3}F$$

$$\sum AH.h_a = \frac{\sum a^2}{2} \le \frac{9R^2}{2} \ (Leibniz)$$

$$Equality\ for\ holds\ a = b = c.$$