ROMANIAN MATHEMATICAL MAGAZINE

If in $\Delta ABC, I_a, I_b, I_c - excenters$ then:

$$[I_aBC] + [I_bAC] + [I_cAB] \geq 3F$$

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$$Let\ a = x + y, b = y + z, c = x + z, s = x + y + z$$

$$Lemma\ 1(Cesaro): (x + y)(y + z)(x + z) \geq 8xyz$$

$$[I_aBC] = \frac{a.r_a}{2}$$

$$LHS = \sum\left(\frac{r_a.a}{2}\right) = \frac{1}{2}\sum\left(\frac{aF}{s - a}\right) = \frac{F}{2}\sum\frac{1}{s - a} = \frac{F}{2}\sum\frac{x + y}{z} \geq$$

$$\geq \frac{3F}{2}.\sqrt[3]{\frac{(x + y)(y + z)(x + z)}{xyz}} \geq \frac{3F}{2}\sqrt[3]{\frac{8xyz}{xyz}} = \frac{3F}{2}.2 = 3F$$

$$Equality\ holds\ for\ a = b = c.$$