ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc}(cosA+cosB)^2+2\sum_{cyc}cosA\leq 6$$

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Solution by Tapas Das-India

$$\sum_{cyc}(cosA+cosB)^2+2\sum_{cyc}cosA=\sum_{cyc}\left(2\cos\frac{A+B}{2}.\cos\frac{A-B}{2}\right)^2+2\sum_{cyc}cosA\leq$$

$$\overset{\cos\frac{A-B}{2}\leq 1}{\leq}\sum_{cyc}\left(2\cos\frac{\pi-C}{2}\right)^2+2\sum_{cyc}cosA=4\sum\sin^2\left(\frac{C}{2}\right)+2\sum_{cyc}cosA=$$

$$=2\sum 2\sin^2\left(\frac{C}{2}\right)+2\sum_{cyc}cosA=2\sum(1-cosC)+2\sum_{cyc}cosA=$$

$$=6-2\sum cosA+2\sum_{cyc}cosA=6$$

Equality holds for an equilateral triangle.