ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1+\cot A}{\sin A}+\frac{1+\cot B}{\sin B}+\frac{1+\cot C}{\sin C}\geq 2+2\sqrt{3}$$

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Solution by Tapas Das-India

$$Let\ x=\cot A, y=\cot B, z=\cot C\ then \sum xy=\sum cotA.cotB=1\ (1)$$

$$\frac{1+\cot A}{\sin A}+\frac{1+\cot B}{\sin B}+\frac{1+\cot C}{\sin C}=\sum\frac{1+\cot A}{\sin A}=$$

$$=\sum(1+cotA)\sqrt{1+\cot^2 A}=\sum(1+x)\sqrt{(1+x^2)}=$$

$$=\sum\sqrt{(1+x^2)}+\sum\sqrt{(x^2+x^4)}\overset{Minkowski}{\geq}$$

$$\geq\sqrt{(1+1+1)^2+(x+y+z)^2}+\sqrt{(x+y+z)^2+(x^2+y^2+z^2)^2}$$

$$\geq\sqrt{9+3(xy+yz+zx)}+\sqrt{3(xy+yz+zx)+(xy+y+zx)^2}\overset{(1)}{\geq}$$

$$\geq\sqrt{9+3}+\sqrt{3+1}=2+2\sqrt{3}$$

Equality holds for an equilateral triangle.