ROMANIAN MATHEMATICAL MAGAZINE

If I −incenter in ΔABC then holds:

$$\frac{1}{AI^2}+\frac{1}{BI^2}+\frac{1}{CI^2}\geq\frac{3}{4r^2}$$

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Solution by Daniel Sitaru-Romania

$$\frac{1}{AI^2}+\frac{1}{BI^2}+\frac{1}{CI^2}=\sum_{cyc}\frac{1}{AI^2}=\sum_{cyc}\frac{1}{\frac{r^2}{\sin^2\frac{A}{2}}}=\frac{1}{r^2}\sum_{cyc}\sin^2\frac{A}{2}=$$

$$=\frac{1}{r^2}\left(1-\frac{r}{2R}\right)\overset{EULER}{\geq}\frac{1}{r^2}\left(1-\frac{\frac{R}{2}}{2R}\right)=\frac{1}{r^2}\left(1-\frac{1}{4}\right)=\frac{3}{4r^2}$$

Equality holds for an equilateral triangle.