ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$a^2\cos\frac{A}{2}+b^2\cos\frac{B}{2}+c^2\cos\frac{C}{2}\geq 6F$$

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Solution by Daniel Sitaru-Romania

$$a^2\cos\frac{A}{2}+b^2\cos\frac{B}{2}+c^2\cos\frac{C}{2}\overset{CEBYSHEV}{\geq}\frac{1}{3}\sum_{cyc}a^2\cdot\sum_{cyc}\cos\frac{A}{2}\geq$$

$$\overset{IONESCU-WEITZENBOCK}{\geq}\frac{1}{3}\cdot 4\sqrt{3}F\cdot\sum_{cyc}\cos\frac{A}{2}\overset{JENSEN}{\geq}\frac{1}{3}\cdot 4\sqrt{3}F\cdot 3\cos\left(\frac{\frac{A}{2}+\frac{B}{2}+\frac{C}{2}}{3}\right)=$$

$$=4\sqrt{3}F\cos\left(\frac{\pi}{6}\right)=4\sqrt{3}F\cdot\frac{\sqrt{3}}{2}=6F$$

Equality holds for an equilateral triangle.