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In ΔABC the following relationship holds:

$$\frac{\sin\left(\frac{A}{2}\right)}{\sin^2(B)}+\frac{\sin\left(\frac{B}{2}\right)}{\sin^2(C)}+\frac{\sin\left(\frac{C}{2}\right)}{\sin^2(A)}\geq 2$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{\sin\left(\frac{A}{2}\right)}{\sin^2(B)}\overset{AM-GM}{\geq}3\sqrt[3]{\frac{\prod_{cyc}\sin\left(\frac{A}{2}\right)}{\left(\prod_{cyc}\sin(A)\right)^2}}=3\sqrt[3]{\frac{\prod_{cyc}\sin\left(\frac{A}{2}\right)}{64\prod_{cyc}\left(\sin\left(\frac{A}{2}\right).\cos\left(\frac{A}{2}\right)\right)^2}}=$$

$$=3\sqrt[3]{\frac{1}{8\left(\prod_{cyc}\sin(A)\right)\left(\prod_{cyc}\cos\left(\frac{A}{2}\right)\right)}}=\frac{3}{2}\sqrt[3]{\frac{1}{\frac{F}{2R^2}\cdot\frac{p}{4R}}}=$$

$$=\frac{3}{2}\sqrt[3]{\frac{8R^3}{F.p}}=3\sqrt[3]{\frac{R.R^2}{r.p^2}}\overset{Euler\ (R\geq 2r)}{\geq}3\sqrt[3]{2.\frac{4p^2}{27p^2}}=2$$

$$Equality\ \ holds\ \ for\ \ A=B=C$$