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In ΔABC the following relationship holds:

$$\frac{ctg^2(A)}{\sin(A)}+\frac{ctg^2(B)}{\sin(B)}+\frac{ctg^2(C)}{\sin(C)}\ge\frac{2\sqrt{3}}{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$\sum_{cyc}\frac{ctg^2(A)}{\sin(A)}\overset{Bergstrom}{\ge}\frac{\left(\sum_{cyc}ctg\,(A)\right)^2}{\sum_{cyc}sin\,(A)}=\frac{\left(\frac{s^2-4Rr-r^2}{2sr}\right)^2}{\frac{s}{R}}\overset{Gerretsen}{\ge}$$

$$\ge\frac{(16Rr-5r^2-4Rr-r^2)^2}{4s^2r^2}\cdot\frac{R}{s}=\frac{(12Rr-6r^2)^2}{4s^2r^2}\cdot\frac{R}{s}=$$

$$=\frac{R.(6R-3r)^2}{s^3}\overset{Euler}{\ge}\frac{R.(6R-1.5R)^2}{s^3}\overset{Mitrinovic}{\ge}\frac{8R.20.25R^2}{81\sqrt{3}R^3}=\frac{2\sqrt{3}}{3}$$

$$Equality\ \ holds\ \ for\ \ A=B=C.$$