ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC the $following$ $relationship$ $holds$:

$$\cos 2A + \cos 2B - \cos 2C \leq \frac{3}{2}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\cos 2A + \cos 2B - \cos 2C = -2\cos C\cos(A-B) - 2\cos^2 C + 1 \overset{?}{\leq} \frac{3}{2}$$

$$\Leftrightarrow \cos^2 C + \cos C\cos(A-B) + \frac{1}{4} \overset{?}{\geq} 0$$

$$\Leftrightarrow \cos^2 C + \cos C\cos(A-B) + \frac{\cos^2(A-B)}{4} + \frac{1-\cos^2(A-B)}{4} \overset{?}{\geq} 0$$

$$\Leftrightarrow \left(\cos C + \frac{\cos(A-B)}{2}\right)^2 + \frac{1-\cos^2(A-B)}{4} \overset{?}{\geq} 0 \to \text{true} \because 1 \geq \cos^2(A-B),$$

$$'' ='' \text{ iff } A = B \wedge \cos C = -\frac{\cos(A-B)}{2} = -\frac{1}{2} \text{ and so,}$$

$$\cos 2A + \cos 2B - \cos 2C \leq \frac{3}{2}, '' ='' \text{ iff } \left(A = B = \frac{\pi}{6}; C = \frac{2\pi}{3}\right)$$