# ROMANIAN MATHEMATICAL MAGAZINE

**In any $\Delta$ ABC the following relationship holds :**

$$\frac{20}{9}(m_a m_b + m_b m_c + m_c m_a) > ab + bc + ca$$

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$$5\sum_{cyc} ab \overset{?}{>} 4\sum_{cyc} m_a m_b \Leftrightarrow 5(s^2+4Rr+r^2) \underbrace{\overset{?}{>}}_{(*)} 2\left(\sum_{cyc} m_a\right)^2 - 2\sum_{cyc} m_a^2 \text{ and}$$

$$\because \left(\sum_{cyc} m_a\right)^2 \overset{\text{Chu-Yang}}{\leq} 4s^2 - 16Rr + 5r^2 \therefore \text{ in order to prove } (*), \text{it suffices}$$

$$\text{to prove : } 5(s^2+4Rr+r^2) + 3(s^2-4Rr-r^2) \overset{?}{>} 2(4s^2-16Rr+5r^2)$$

$$\Leftrightarrow 8(5Rr - r^2) \overset{?}{>} 0 \to \text{true} \because 5Rr \overset{\text{Euler}}{\geq} 10r^2 > r^2 \Rightarrow (*) \text{ is true}$$

$$\therefore 5\sum_{cyc} ab > 4\sum_{cyc} m_a m_b \text{ and implementing it on a triangle with sides :}$$

$$\frac{2m_a}{3}, \frac{2m_b}{3}, \frac{2m_c}{3}, \text{whose medians as a consequence of trivial calculations} =$$

$$\frac{a}{2}, \frac{b}{2}, \frac{c}{2} \text{ respectively, we get : } 5.\frac{4}{9}\sum_{cyc} m_a m_b > \frac{4}{4}\sum_{cyc} ab$$

$$\Rightarrow \frac{20}{9}(m_a m_b + m_b m_c + m_c m_a) > ab + bc + ca \ \forall \Delta \text{ ABC (QED)}$$