ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1+\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}+\frac{1+\sin\left(\frac{B}{2}\right)}{\cos\left(\frac{B}{2}\right)}+\frac{1+\sin\left(\frac{C}{2}\right)}{\cos\left(\frac{C}{2}\right)}\geq 3\sqrt{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$Let\ :\ f(x)=\frac{1+\sin(x)}{\cos(x)}\ \ and\ \ x\in\left(0;\frac{\pi}{2}\right)\ Then\ we\ get$$

$$f'(x)=\frac{1+\sin(x)}{cos^2(x)}\ ,\qquad f''(x)=\left(\frac{1+\sin(x)}{cos^2(x)}\right)'=$$

$$=\frac{(1+\sin(x))'.cos^2(x)-(1+\sin(x))\left(cos^2(x)\right)'}{cos^4(x)}=\frac{(1+\sin(x))^2}{cos^3(x)}>0$$

$$The\ function\ \ f(x)=\frac{1+\sin(x)}{\cos(x)}\ \ satisfies\ the\ conditions\ of$$

$$Jensen's\ inequality,\qquad f(x)\ is\ a\ convex\ function.$$

$$\sum_{cyc}\frac{1+\sin\left(\frac{A}{2}\right)}{\cos\left(\frac{A}{2}\right)}\geq 3.\frac{1+\sin\left(\frac{A+B+C}{6}\right)}{\cos\left(\frac{A+B+C}{6}\right)}=3.\frac{1+\sin\left(\frac{\pi}{6}\right)}{\cos\left(\frac{\pi}{6}\right)}=3.\frac{1+\frac{1}{2}}{\frac{\sqrt{3}}{2}}=3\sqrt{3}$$

$$Equality\ \ holds\ \ for\ \ A=B=C.$$