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If I –incenter in $\triangle ABC$ then:

$$\frac{1}{AI^2} + \frac{1}{BI^2} + \frac{1}{CI^2} \geq \frac{3}{4r^2}$$

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Solution by Daniel Sitaru-Romania

$$\begin{aligned} \frac{1}{AI^2} + \frac{1}{BI^2} + \frac{1}{CI^2} &= \sum_{cyc} \frac{1}{AI^2} = \sum_{cyc} \frac{1}{r^2 \frac{\sin^2 \frac{A}{2}}{\sin^2 \frac{A}{2}}} = \frac{1}{r^2} \sum_{cyc} \sin^2 \frac{A}{2} = \\ &= \frac{1}{r^2} \left(1 - \frac{r}{2R}\right) \stackrel{EULER}{\geq} \frac{1}{r^2} \left(1 - \frac{\frac{R}{2}}{2R}\right) = \frac{1}{r^2} \left(1 - \frac{1}{4}\right) = \frac{3}{4r^2} \end{aligned}$$

Equality holds for $a = b = c$.