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In ΔABC the following relationship holds:

$$\frac{a^2}{w_a r_a}+\frac{b^2}{w_b r_b}+\frac{c^2}{w_c r_c}\geq 4$$

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$$w_a=\frac{2bc}{b+c}\cos\left(\frac{A}{2}\right), r_a=s\cdot\tan\left(\frac{A}{2}\right)\ and\ \sin\left(\frac{A}{2}\right)=\sqrt{\frac{(s-b)(s-c)}{bc}}$$

$$w_a\cdot r_a=\frac{2bcs}{b+c}\cdot\sin\left(\frac{A}{2}\right)=\frac{2s\sqrt{bc(s-b)(s-c)}}{b+c}$$

$$\frac{b+c}{2}\overset{AM-GM}{\geq}\sqrt{bc}\rightarrow\frac{2\sqrt{bc}}{b+c}\leq 1$$

$$w_a r_a\leq s\sqrt{(s-b)(s-c)}\rightarrow\sqrt{(s-b)(s-c)}\overset{AM-GM}{\leq}\frac{2s-(b+c)}{2}=\frac{a}{2}$$

$$w_a r_a\leq\frac{as}{2}\rightarrow\frac{1}{w_a r_a}\geq\frac{2}{as}\rightarrow\frac{a^2}{w_a r_a}\geq\frac{2a}{s}$$

$$\sum_{cyc}\frac{a^2}{w_a r_a}\geq\frac{2}{s}(a+b+c)=\frac{4s}{s}=4$$

Equlity holds for $a=b=c$.