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In ΔABC the following relationship holds:

$$\frac{r_a}{b}+\frac{r_b}{c}+\frac{r_c}{a}\geq\frac{h_a}{b}+\frac{h_b}{c}+\frac{h_c}{a}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{h_a}{b}+\frac{h_b}{c}+\frac{h_c}{a}=\frac{2F}{ab}+\frac{2F}{cb}+\frac{2F}{ac}=$$

$$\frac{2F(a+b+c)}{abc}=\frac{4F.s}{4F.R}=s$$

$$\frac{r_a}{b}+\frac{r_b}{c}+\frac{r_c}{a}\overset{AM-GM}{\geq}3\left(\frac{r_ar_br_c}{abc}\right)^{\frac{1}{3}}=3\left(\frac{rs^2}{4RF}\right)^{\frac{1}{3}}=$$

$$3\left(\frac{rs^2}{4R.rs}\right)^{\frac{1}{3}}=3\left(\frac{s}{4R}\right)^{\frac{1}{3}}$$

$$Let's\ prove\ that:$$

$$3\left(\frac{s}{4R}\right)^{\frac{1}{3}}\geq\frac{s}{R}\to\frac{27}{4}.\frac{s}{R}\geq\left(\frac{s}{R}\right)^3\to$$

$$\frac{27}{4}\geq\left(\frac{s}{R}\right)^2\to s\leq\frac{3\sqrt{3}}{2}R\ (Mitrinovic)\ True$$

$$Equality\ holds\ for\ a=b=c.$$