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In ΔABC the following relationship holds:

$$\frac{ctg\left(\frac{A}{2}\right)}{ctg^2\left(\frac{B}{2}\right)}+\frac{ctg\left(\frac{B}{2}\right)}{ctg^2\left(\frac{C}{2}\right)}+\frac{ctg\left(\frac{C}{2}\right)}{ctg^2\left(\frac{A}{2}\right)}\geq\sqrt{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mirsadix Muzefferov-Azerbaijan

$$\frac{ctg\left(\frac{A}{2}\right)}{ctg^2\left(\frac{B}{2}\right)}+\frac{ctg\left(\frac{B}{2}\right)}{ctg^2\left(\frac{C}{2}\right)}+\frac{ctg\left(\frac{C}{2}\right)}{ctg^2\left(\frac{A}{2}\right)}\overset{AM-GM}{\geq}3\left(\frac{ctg\left(\frac{A}{2}\right)}{ctg^2\left(\frac{B}{2}\right)}\cdot\frac{ctg\left(\frac{B}{2}\right)}{ctg^2\left(\frac{C}{2}\right)}\cdot\frac{ctg\left(\frac{C}{2}\right)}{ctg^2\left(\frac{A}{2}\right)}\right)^{\frac{1}{3}}=$$

$$=3\left(\prod_{cyc}\tan\left(\frac{A}{2}\right)\right)^{\frac{1}{3}}=3\left(\frac{r}{s}\right)^{\frac{1}{3}}\overset{Mitrinovic}{\geq}3\left(\frac{1}{3\sqrt{3}}\right)^{\frac{1}{3}}=\sqrt{3}$$

$$Equality\ \ holds\ \ for\ \ A=B=C.$$