If $I-$incenter in $\triangle ABC$ then the following relationship holds:

$$\frac{1}{IA} + \frac{1}{IB} + \frac{1}{IC} \geq \frac{3}{R}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{1}{IA} + \frac{1}{IB} + \frac{1}{IC} = \sum_{cyc} \frac{1}{IA} = \sum_{cyc} \frac{sin\frac{A}{2}}{r} = \frac{1}{r}\sum_{cyc} sin\frac{A}{2} \overset{JENSEN}{\geq}$$

$$\geq \frac{1}{r} \cdot 3sin\left(\frac{A+B+C}{6}\right) = \frac{6}{2r} \cdot sin\frac{\pi}{3} \overset{EULER}{\geq} \frac{6}{R} \cdot \frac{1}{2} = \frac{3}{R}$$

Equality holds for: $a = b = c$.