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In ΔABC the following relationship holds:

$$\frac{1}{w_a}+\frac{1}{w_b}+\frac{1}{w_c}\ge\frac{2}{R}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{1}{w_a}+\frac{1}{w_b}+\frac{1}{w_c}\overset{AM-GM}{\ge}\frac{3}{\sqrt[3]{w_aw_bw_c}}\ge\frac{3}{\sqrt[3]{r_ar_br_c}}=$$

$$=\frac{3}{\sqrt[3]{\frac{F}{s-a}\cdot\frac{F}{s-b}\cdot\frac{F}{s-c}}}=\frac{3}{\sqrt[3]{\frac{F^3}{(s-a)(s-b)(s-c)}}}=\frac{3}{\sqrt[3]{\frac{F^3s}{s(s-a)(s-b)(s-c)}}}=\frac{3}{\sqrt[3]{\frac{F^3s}{F^2}}}=\frac{3}{\sqrt[3]{Fs}}=\frac{3}{\sqrt[3]{rs^2}}$$

$$\frac{3}{\sqrt[3]{rs^2}}\ge\frac{2}{R}\Leftrightarrow\frac{27}{rs^2}\ge\frac{8}{R^3}\Leftrightarrow 7R^3\ge 8rs^2\ (to\ prove)$$

$$8rs^2\overset{EULER}{\le}4Rs^2\overset{MITRINOVICI}{\le}4R\cdot\left(\frac{3\sqrt{3}R}{2}\right)^2=27R^3$$

Equality holds for $a=b=c$.