ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\frac{1+sinA}{cosA}+\frac{1+sinB}{cosB}+\frac{1+sinC}{cosC}\ge 6+3\sqrt{3}$$

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Solution by Daniel Sitaru

$$\frac{1+sinA}{cosA}+\frac{1+sinB}{cosB}+\frac{1+sinC}{cosC}=\sum_{cyc}\frac{1+sinA}{cosA}=$$

$$=\sum_{cyc}\frac{1}{cosA}+\sum_{cyc}tanA\overset{JENSEN}{\overbrace{\ge}}\sum_{cyc}\frac{1}{cosA}+3tan\left(\frac{A+B+C}{3}\right)\ge$$

$$\overset{BERGSTROM}{\overbrace{\ge}}\frac{(1+1+1)^2}{cosA+cosB+cosC}+3tan\left(\frac{\pi}{3}\right)=\frac{9}{1+\frac{r}{R}}+3\sqrt{3}=$$

$$=\frac{9R}{R+r}+3\sqrt{3}\overset{EULER}{\overbrace{\ge}}\frac{9R}{R+\frac{R}{2}}+3\sqrt{3}=\frac{9\cdot 2}{3}+3\sqrt{3}=6+3\sqrt{3}$$

Equality holds for $a=b=c$.