ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$\sum_{cyc} \frac{secA}{sinA} \geq 4\sqrt{3}$$

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Solution by Daniel Sitaru-Romania

$$\sum_{cyc} \frac{secA}{sinA} = \sum_{cyc} \frac{1}{sinAcosA} = 2\sum_{cyc} \frac{1}{sin2A} \overset{BERGSTROM}{\geq}$$

$$\geq 2 \cdot \frac{(1+1+1)^2}{sin2A + sin2B + sin2C} = \frac{2 \cdot 9}{4sinAsinBsinC} =$$

$$= \frac{9}{2 \cdot \frac{a}{2R} \cdot \frac{b}{2R} \cdot \frac{c}{2R}} = \frac{36R^3}{abc} = \frac{36R^3}{4RF} = \frac{9R^2}{rs} \overset{EULER}{\geq}$$

$$\geq \frac{9R^2}{\frac{R}{2} \cdot s} = \frac{18R}{s} \overset{MITRINOVICI}{\geq} \frac{18 \cdot \frac{2s}{3\sqrt{3}}}{s} = \frac{12}{\sqrt{3}} = 4\sqrt{3}$$

Equality holds for $a = b = c$.