

ROMANIAN MATHEMATICAL MAGAZINE

In $\triangle ABC$ the following relationship holds:

$$\frac{\sin(A)}{\tan(B) + \tan(C)} + \frac{\sin(B)}{\tan(C) + \tan(A)} + \frac{\sin(C)}{\tan(A) + \tan(B)} \geq \frac{3}{4}$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$\begin{aligned} \sum_{cyc} \frac{\sin(A)}{\tan(B) + \tan(C)} &= \sum_{cyc} \frac{\sin(A)}{\frac{\sin(B+C)}{\cos(B) \cdot \cos(C)}} = \\ &= \sum_{cyc} \cos(A) \cdot \cos(B) = \frac{s^2 - r^2 - 4R^2}{4R^2} \stackrel{\text{Gerretsen}}{\geq} \\ &\leq \frac{4R^2 + 4Rr + 3r^2 + r^2 - R^2}{4R^2} = \frac{Rr + r^2}{R^2} \stackrel{\text{Euler}}{\geq} \frac{\frac{R^2}{2} + \frac{R^2}{4}}{R^2} = \frac{3}{4} \end{aligned}$$

Equality holds for : $A = B = C$.