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In $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} \frac{\cot B + \cot C}{\sin A} \geq 4$$

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$$\begin{aligned} \frac{\cot(B) + \cot(C)}{\sin(A)} &= \frac{\frac{\cos(B)}{\sin(B)} + \frac{\cos(C)}{\sin(C)}}{\sin(A)} = \frac{\sin(B)\cos(C) + \sin(C)\cos(B)}{\sin(A) \cdot \sin(B) \cdot \sin(C)} = \\ &= \frac{\sin(B+C)}{\sin(A) \cdot \sin(B) \cdot \sin(C)} = \frac{1}{\sin(B) \cdot \sin(C)} \\ \sum_{cyc} \frac{1}{\sin(B) \cdot \sin(C)} &= \frac{\sin(A) + \sin(B) + \sin(C)}{\sin(A) \cdot \sin(B) \cdot \sin(C)} = \frac{\frac{s}{R}}{\frac{F}{2R^2}} = \frac{2sR^2}{FR} = \frac{2sR}{sr} = \frac{2R^{Euler}}{r} \stackrel{Euler}{=} 4 \end{aligned}$$

Equality holds for : $A = B = C$