

# ROMANIAN MATHEMATICAL MAGAZINE

In  $\triangle ABC$  the following relationship holds:

$$\sum_{cyc} \csc\left(\frac{A}{2}\right) \left(1 + \sin\left(\frac{A}{2}\right)\right) \geq 9$$

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$$\sum_{cyc} \csc\left(\frac{A}{2}\right) \left(1 + \sin\left(\frac{A}{2}\right)\right) \geq 9$$

$$\sum_{cyc} \csc\left(\frac{A}{2}\right) + \sum_{cyc} \csc\left(\frac{A}{2}\right) \sin\left(\frac{A}{2}\right) \geq 9 \rightarrow \sum_{cyc} \csc\left(\frac{A}{2}\right) + 3 \geq 9$$

$$\sum_{cyc} \csc\left(\frac{A}{2}\right) \geq 6 \rightarrow A + B + C = \pi \quad A \in (0; \pi) \rightarrow \csc\left(\frac{A}{2}\right) > 0$$

$$f(x) = \csc\left(\frac{x}{2}\right) \rightarrow \frac{d^2}{dx^2} f(x) = \frac{1}{4} \csc\left(\frac{x}{2}\right) \left(\csc^2\left(\frac{x}{2}\right) + \operatorname{ctg}^2\left(\frac{x}{2}\right)\right)$$

$$x \in (0; \pi) \rightarrow \frac{d^2}{dx^2} f(x) > 0$$

$$\frac{1}{3} \sum_{cyc} \csc\left(\frac{A}{2}\right) \stackrel{JENSEN}{\geq} \csc\left(\frac{\frac{A}{2} + \frac{B}{2} + \frac{C}{2}}{3}\right) \rightarrow \sum_{cyc} \csc\left(\frac{A}{2}\right) \geq 3 \csc\left(\frac{\pi}{6}\right) = 6$$

$$\sum_{cyc} \csc\left(\frac{A}{2}\right) \left(1 + \sin\left(\frac{A}{2}\right)\right) \geq 9$$

Equality holds for an equilateral triangle.