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In ΔABC the following relationship holds:

$$\frac{\sqrt{1+\tan^2\frac{A}{2}}}{\sin\frac{A}{2}}+\frac{\sqrt{1+\tan^2\frac{B}{2}}}{\sin\frac{B}{2}}+\frac{\sqrt{1+\tan^2\frac{C}{2}}}{\sin\frac{C}{2}}\geq 4\sqrt{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{\sqrt{1+\tan^2\frac{A}{2}}}{\sin\frac{A}{2}}+\frac{\sqrt{1+\tan^2\frac{B}{2}}}{\sin\frac{B}{2}}+\frac{\sqrt{1+\tan^2\frac{C}{2}}}{\sin\frac{C}{2}}=\sum_{cyc}\frac{\sqrt{1+\tan^2\frac{A}{2}}}{\sin\frac{A}{2}}=$$

$$=\sum_{cyc}\frac{\sqrt{\frac{1}{\cos^2\frac{A}{2}}}}{\sin\frac{A}{2}}=\sum_{cyc}\frac{1}{\sin\frac{A}{2}\cos\frac{A}{2}}=\sum_{cyc}\frac{2}{2\sin\frac{A}{2}\cos\frac{A}{2}}=\sum_{cyc}\frac{2}{\sin A}=$$

$$=\sum_{cyc}\frac{4R}{a}=4R\sum_{cyc}\frac{1}{a}\overset{AM-GM}{\geq}4R\cdot\frac{3}{\sqrt[3]{abc}}=\frac{12R}{\sqrt[3]{4Rrs}}\overset{MITRINOVICI}{\geq}$$

$$\geq\frac{12R}{\sqrt[3]{4Rr\cdot\frac{3\sqrt{3}}{2}R}}=\frac{12R}{\sqrt{3}\cdot\sqrt[3]{2R^2r}}\overset{EULER}{\geq}\frac{12R}{\sqrt{3}\cdot\sqrt[3]{2R^2\cdot\frac{R}{2}}}=\frac{12}{\sqrt{3}}=4\sqrt{3}$$

Equality holds for an equilateral triangle.