ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$2R^2(sin^3A+sin^3B+sin^3C)\ge 3F$$

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Solution by Daniel Sitaru-Romania

$$2R^2(sin^3A+sin^3B+sin^3C)=2R^2\sum_{cyc} sin^3A=$$

$$=2R^2\sum_{cyc}\left(\frac{a}{2R}\right)^3=\frac{2R^2}{8R^3}\sum_{cyc}a^3\overset{AM-GM}{\ge}\frac{1}{4R}\cdot 3\sqrt[3]{(abc)^3}=$$

$$=\frac{3abc}{4R}=\frac{3\cdot 4RF}{4R}=3F$$

Equality holds for an equilateral triangle.