ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{1+\sin A}{\cos A}+\frac{1+\sin B}{\cos B}+\frac{1+\sin C}{\cos C}\geq 6+3\sqrt{3}$$

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$$f:\left(0,\frac{\pi}{2}\right)\to\mathbb{R}, f(x)=\frac{1+\sin x}{\cos x}=\frac{1}{\cos x}+\tan x$$

$$f'(x)=\frac{1+\sin x}{\cos^2 x}, f''(x)=\frac{\sin^2 x+2\sin x+1}{\cos^2 x}\geq 0\Rightarrow$$

f −*convexe. By Jensen's inequality:*

$$\sum_{cyc}\frac{1+\sin A}{\cos A}=\sum_{cyc}f(A)\geq 3f\left(\frac{A+B+C}{3}\right)=3f\left(\frac{\pi}{3}\right)=$$

$$=3\cdot\frac{1+\sin\frac{\pi}{3}}{\cos\frac{\pi}{3}}=3\cdot\frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}=6+3\sqrt{3}$$

Equality holds for an equilateral triangle.