ROMANIAN MATHEMATICAL MAGAZINE

If I –incenter then in ΔABC the following relationship holds:

$$IA \cdot IB \cdot IC \leq R^3$$

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Solution by Daniel Sitaru-Romania

$$IA \cdot IB \cdot IC = \prod_{cyc} IA = \prod_{cyc} \frac{r}{\sin\frac{A}{2}} =$$

$$= r^3 \cdot \frac{1}{\sqrt{\frac{(s-b)(s-c)}{bc}} \cdot \sqrt{\frac{(s-c)(s-a)}{ca}} \cdot \sqrt{\frac{(s-a)(s-b)}{ab}}} =$$

$$= \frac{r^3}{\frac{(s-a)(s-b)(s-c)}{abc}} = \frac{abcsr^3}{s(s-a)(s-b)(s-c)} = \frac{4RFsr^3}{F^2} =$$

$$= \frac{4Rsr^3}{F} = \frac{4Rsr^3}{rs} = 4Rr^2 \overset{EULER}{\leq} 4R \cdot \frac{R^2}{4} = R^3$$

Equality holds for an equilateral triangle.