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If I −incenter in ΔABC then:

$$\frac{m_a^2}{IA^2}+\frac{m_b^2}{IB^2}+\frac{m_c^2}{IC^2}\ge\frac{27}{4}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\frac{m_a^2}{IA^2}+\frac{m_b^2}{IB^2}+\frac{m_c^2}{IC^2}=\sum_{cyc}\frac{m_a^2}{IA^2}=\sum_{cyc}\frac{m_a^2}{\frac{r^2}{\sin^2\frac{A}{2}}}=\frac{1}{r^2}\sum_{cyc}m_a^2\sin^2\frac{A}{2}\overset{CEBYSHEV}{\ge}$$

$$\ge\frac{1}{3r^2}\left(\sum_{cyc}m_a^2\right)\left(\sum_{cyc}\sin^2\frac{A}{2}\right)=\frac{1}{3r^2}\cdot\frac{3}{4}\sum_{cyc}a^2\cdot\left(1-\frac{r}{2R}\right)\ge$$

$$\overset{IONESCU-WEITZENBOCK}{\ge}\frac{1}{4r^2}\cdot 4\sqrt{3}F\cdot\left(1-\frac{r}{2R}\right)\overset{EULER}{\ge}\frac{1}{4r^2}\cdot 4\sqrt{3}rs\cdot\left(1-\frac{\frac{R}{2}}{2R}\right)=$$

$$=\frac{\sqrt{3}s}{r}\cdot\left(1-\frac{1}{4}\right)\overset{MITRINOVIC}{\ge}\frac{\sqrt{3}\cdot 3\sqrt{3}r}{r}\cdot\frac{3}{4}=\frac{27}{4}$$

Equality holds for an equilateral triangle.