

In acute $\triangle ABC$ the following relationship holds:

$$\sum_{cyc} \frac{\sin \frac{A}{2}}{\left(1 + \cot^2 \frac{A}{2}\right)^2} \geq \frac{3}{32}$$

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$$\begin{aligned} \sum \sin^2 \frac{A}{2} &= 1 - \frac{r}{2R} \stackrel{\text{Euler}}{\geq} 1 - \frac{1}{4} = \frac{3}{4} \\ \sum_{cyc} \frac{\sin \frac{A}{2}}{\left(1 + \cot^2 \frac{A}{2}\right)^2} &= \sum \frac{\sin \frac{A}{2}}{\left(\csc^2 \frac{A}{2}\right)^2} = \sum \sin^5 \frac{A}{2} = \\ &= \sum \left(\sin^2 \frac{A}{2}\right)^{\frac{5}{2}} \stackrel{\text{CBS}}{\geq} 3 \cdot \left(\frac{\sum \sin^2 \frac{A}{2}}{3}\right)^{\frac{5}{2}} \geq 3 \cdot \left(\frac{\frac{3}{4}}{3}\right)^{\frac{5}{2}} = \frac{3}{32} \end{aligned}$$

Equality holds for an equilateral triangle.