In acute ΔABC the following relationship holds:

$$\sum_{cyc} \frac{cosA}{(1+tan^2A)^2} \geq \frac{3}{32}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Tapas Das-India

$$\sum cos^2A = 3 - \sum sin^2A = 3 - \frac{\sum a^2}{4R^2} \overset{Leibniz}{\geq} 3 - \frac{9R^2}{4R^2} = \frac{3}{4}$$

$$\sum_{cyc} \frac{cosA}{(1+tan^2A)^2} = \sum \frac{\cos A}{.(\sec^2 A)^2} = \sum cos^5A =$$

$$= \sum (cos^2A)^{\frac{5}{2}} \overset{CBS}{\geq} 3.\left(\frac{\sum cos^2A}{3}\right)^{\frac{5}{2}} \geq 3.\left(\frac{\frac{3}{4}}{3}\right)^{\frac{5}{2}} = \frac{3}{32}$$

Equality holds for an equilateral triangle.