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In ΔABC the following relationship holds:

$$\frac{\cos\frac{A}{2}}{\sin\frac{B}{2}}+\frac{\cos\frac{B}{2}}{\sin\frac{C}{2}}+\frac{\cos\frac{C}{2}}{\sin\frac{A}{2}}\ge 3\sqrt{3}$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Daniel Sitaru-Romania

$$\sum_{cyc}\frac{\cos\frac{A}{2}}{\sin\frac{B}{2}}\overset{AM-GM}{\ge}3\sqrt[3]{\frac{\cos\frac{A}{2}}{\sin\frac{B}{2}}\frac{\cos\frac{B}{2}}{\sin\frac{C}{2}}\cdot\frac{\cos\frac{C}{2}}{\sin\frac{A}{2}}}=3\sqrt[3]{\frac{s\sqrt{s(s-a)(s-b)(s-c)}}{(s-a)(s-b)(s-c)}}=$$

$$3\sqrt[3]{\frac{s^2F}{F^2}}=3\sqrt[3]{\frac{s^2}{rs}}=3\sqrt[3]{\frac{s}{r}}\overset{MITRINOVICI}{\ge}3\sqrt[3]{\frac{3\sqrt{3}r}{r}}=3\sqrt[3]{\left(\sqrt{3}\right)^3}=3\sqrt{3}$$

Equality holds for $A=B=C$.