ROMANIAN MATHEMATICAL MAGAZINE

Prove that in any non isosceles triangle ABC holds:

$$\sqrt{\sum_{cyc}\frac{4(a^2-ab+b^2)}{(a-b)^2}}>3$$

Proposed by Neculai Stanciu-Romania

Solution by Tapas Das-India

$$4(a^2-ab+b^2)=2\times 2(a^2+b^2)-4ab=$$
$$=2((a+b)^2+(a-b)^2)-((a+b)^2-(a-b)^2)=(a+b)^2+3(a-b)^2$$

$$\frac{4(a^2-ab+b^2)}{(a-b)^2}=\frac{(a+b)^2+3(a-b)^2}{(a-b)^2}=\frac{(a+b)^2}{(a-b)^2}+3>3\ (1)$$

$$\sqrt{\sum_{cyc}\frac{4(a^2-ab+b^2)}{(a-b)^2}}\overset{(1)}{>}\sqrt{3+3+3}=3$$